

The Price Theory of Two-Sided Markets*

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Abstract

I show that the comparative statics of the Rochet and Tirole (2003) model of two-sided markets depend crucially on the rate at which the firm passes through the cross-subsidies from one side of the market to the other. If this is less than 1 on both sides, I obtain a formal demonstration of the conventional wisdom (Evans, 2003; Wright, 2004) on policy in two-sided markets. Conversely if pass-through is greater than 1 on one side, these policy implications are reversed. Exogenous cost variations identify the pass-through rate and can be used to test the model.

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1 Introduction

With the growth of the “New Economy” in the 1990’s, antitrust and regulatory attention has increasingly turned to high tech industries. Many of these (payment systems, buyer-to-buyer services, video game consoles, etc.) act as platforms over which two groups of users interact. For example, internet service providers allow web surfers to view content supplied by content providers, like Google and Yahoo!. In response to this policy interest, a burgeoning literature on so-called two-sided markets has grown up in recent years. Progress in modeling two-sided markets (Rochet and Tirole, 2003; Armstrong, 2006; Caillaud and Jullien, 2001, 2003) has cautioned policy makers against the naïve application of old price theory lenses to these new markets (Wright, 2004). The main policy message coming out of this pioneering work has been warnings (Kind et al., 2007; Fahri and Hagiu, 2007; Hagiu and Jullien, 2008) about the surprising effects that are possible in two-sided markets, rather than an alternative framework for regulatory and antitrust policy. Furthermore, the common and more comprehensive policy conclusions based on this literature (Evans, 2003; Wright, 2004; Roson, 2005) have not been formally established in any model. This paper aims to begin bridging this gap, building a framework for policy-oriented empirical analysis on the foundations of the Rochet and Tirole (2003) (RT2003) model of two-sided markets.

While admittedly special to the RT2003 setting, this framework allows empirical data to shed light on policy: many policy-relevant questions can be answered non-parametrically using data including exogenous cost variations. Furthermore, under monopoly, the validity of the model can be tested using the same data, partially allaying fears of misspecification.

Consider a monopolistic credit card company that charges a (possibly negative) per-transaction fee to both merchants and card-holders. The more it charges merchants, the greater incentive it has to persuade consumers to use their cards: the merchant’s price acts as a subsidy to the firm in serving consumers. In fact the defining feature of the RT2003 model is that this cross-subsidy is the *only* cross-effect in the firm’s pricing to the two sides. This makes the rate at which the firm passes these cross-subsidies on to consumers on the

other side of the market crucial. Furthermore this *pass-through rate* is closely tied to the external benefits consumers derive when an additional merchant decides to accept cards. In fact the sign of nearly all comparative statics of interest in the RT2003 model turn on whether or not pass-through is less than one-for-one on both sides of the market.

If it is, I obtain a formal proof of the “conventional wisdom”, which I discuss in Section 2, that the *price level*, the sum of price charged to the two sides, rather than the *price balance*, how this total price is divided between the two sides, should be the focus of policy. If it is not, the conventional wisdom is almost entirely undermined. To motivate these results, Section 3 and 4 briefly review the notion of pass-through in a standard market and the RT2003 model respectively. Section 5 treats the effects of changes in industrial organization and policy interventions on prices, showing that the distinction between the two cases described above can be non-parametrically identified with data on exogenous cost variations, while at the same time allowing the model to be tested if the industry is monopolized. Section 6 considers normative issues, showing how the role of pass-through in determining average surplus also bisects normative issues into the same two cases. Section 7 concludes by discussing directions for future research. Technicalities are worked out in appendices following the main text.

2 Two-Sided Markets and the Conventional Wisdom

Following the definition given by Rochet and Tirole (2006) a two-sided market is a network industry (Katz and Shapiro, 1985) with two distinct groups of consumers where

1. Most or all of the network effects are *across* rather than *within* groups.
2. The division of pricing between the two groups, and not merely some notion of the overall price charged to the groups, matters for the welfare of both groups and the profits earned by the firms serving them.

Typical examples of the first condition are firms serving as a platform for transactions (payment cards), two-sided services (advertising, website access, video game playing) or

matching (dating clubs or websites). For the second condition to be satisfied, as observed by Rochet and Tirole (2006) (RT2006), it is necessary that frictions (legal, social, informational or strategic) prevent the sides from reaching a Coasian bargain that renders the division of prices between the two sides neutral. In fact the classic approaches of RT2003 and Armstrong (2006) in the literature do not model directly the interaction between the two sides, instead positing exogenous demand functions on each side of the market and assuming that the value to consumers on each side of the market is linear in the number of consumers participating on the other side of the market. Thus each side brings external benefits to the other by participating.

Since its beginnings, the two-sided markets literature has given much attention to the distinction between the so-called “price level” and the “price balance” or “structure”. The first has been used extensively in an informal way (Evans, 2003; Wright, 2004; Roson, 2005) to refer to some sense of the overall price charged to the two sides and has been formally defined by Rochet and Tirole (2006) (RT2006) as the sum of the per-interaction prices charged to the two sides of the market. The second usually refers to the way in which this total price is divided between consumers on the two sides of the market. A major contention of the two-sided markets literature has been that policy makers should focus on price level, rather than its balance. First, from a normative perspective, it is argued, in the words of Rochet and Tirole (2006), that “price structures are less likely to be distorted...than the price levels”. Second, on the positive side, it is argued that standard price theory intuitions about the effects of interventions and changes in industrial structure, such as competition, price controls, taxes etc. are more valid with regard to the price level than the balance. For example Evans (2003) writes, “Not surprisingly the price level tends to be lower with...the availability of substitutes (which) tends to put pressure on the two-sided firms to lower their prices” and Wright (2004) writes “While competition will generally lower the total (or average) level of prices charged to men and women, it will not necessarily lower the price charged to men relative to women.” That is, while competition and price controls may be

relied upon to reduce the price level, it is not clear what their effects will be on the balance of prices or even whether they will necessarily reduce prices on both sides of the market. These views together constitute what I call the “conventional wisdom” on two-sided markets.

However, the sense in which these arguments are formally true has been unclear. In fact the primitive notions on which they are based do not have much meaning in one of the two most prominent models of two-sided markets, that of Armstrong (2006). The Armstrong model shares the RT2003 model’s assumption that consumers valuation of interactions with the other side of the market is linear in the number of participants on the other side of the market. However, at this point they diverge. Armstrong assumes that all costs to the firm are per-consumer rather than per-interaction and that all prices to consumers are fixed (one price for membership in the platform, entitling the member to interaction with all participating partners). Furthermore, he assumes that all consumers have the same (linear) valuation of partners on the other side of the market and therefore that any consumer heterogeneity is in their fixed value of joining the platform. In this context, the significance of the price level and balance of RT2006 are unclear; while they can be formally defined, it is not obvious why one should care about the per-transaction prices to the two sides when most of their valuation (or at least their heterogeneity) is interaction-independent. Furthermore it is not at all clear that economically meaningful conditions imply relevant comparative statics with respect to these variables in Armstrong’s model.

On the other hand, in the RT2003 model these notions have a much stronger foundation. RT2003 assume that all costs to the firm, all prices¹ and all consumer valuations (and therefore heterogeneity) are per-interaction. In this model the intuitive meaning and importance of the price level and price balance are clear. Therefore if the conventional wisdom has validity anywhere, it must be in the RT2003 model. Nonetheless even here its status is unclear: no formal demonstration of any results underlying the conventional wisdom has been given. Furthermore numerous claims have been made in the literature that elements of the conven-

¹This distinction matters only in the competitive version of the model.

tional wisdom fail even in the RT2003 model. For example in a model closely related to that of RT2003, Chakravorti and Roson (2006) claim that competition unambiguously reduces prices to both groups of consumers², contradicting a primary contention of the conventional wisdom. Economides and Tåg (2007) claim that price caps on consumers on one side of the market unambiguously improves consumer and social welfare, while the conventional wisdom cautions that interventions to reduce prices on one side of the market may have unexpectedly negative effects. Many authors have also claimed (Bolt and Tieman, 2005; DeGrauwe et al., 2007) as Roson (2005) puts it, “an increase in elasticity in one sub-market *increases* the specific relative price” (emphasis his), which contradicts the conventional wisdom claim that competition for one group of consumers tends to lower their prices while raising prices to the other group of consumers.

This paper tries to understand when and to what extent the conventional wisdom is the appropriate framework for policy and when it is not, what is. In the “regular case”, when demand on both sides of the market are log-concave as was assumed in RT2003 and (as far as I know) all work in this model until now, I confirm and clarify the conventional wisdom. Competition, price controls and subsidies always lower the price level, but their effects on the price balance and even the direction of their effect on individual prices depends on the details of the intervention and market conditions. Interventions that are balanced across the two sides of the market (such as subsidies, competition of equal intensity on the two sides and controls on the price level) reduce prices to both groups of consumers. On the normative side

1. Any change in the price balance away from the monopoly optimum will hurt average consumers on at least one side of the market.
2. The price balance chosen by the monopolist may well be optimal and even when it is not it may be difficult to determine which direction it would be beneficial for it move.

²A counterexample to their proof is available on request.

3. The price level is optimally *below* cost because of externalities, even under monopoly governance of the price balance.
4. Only balanced interventions can be guaranteed to improve consumer welfare.

However when demand is log-convex on one side of the market, which is possible as the problem's second-order conditions only require that one demand be log-concave, things can be entirely different. On the positive side, competition and price controls may well raise the price level and balanced interventions tend to systematically shift the price balance in favor of the log-concave side. The normative picture changes entirely as well:

1. Price balance can always be shifted to benefit (average) consumers on both sides of the market.
2. Simple empirical tests can determine the appropriate direction of this shift.
3. The optimal price level under monopoly governance of the balance may well be above cost.
4. A rise in cost may benefit consumers.

The distinction between these cases can be identified with exogenous cost variations and, under monopoly, the model can even be tested. The key to these results is the importance of the pass-through rate, which I now review in the context of a standard market.

3 Preliminaries

3.1 Pass-Through

A monopolist faces consumer demand³ $D(\cdot)$ and linear cost c . The familiar first-order condition is

³Assumed thrice continuously differentiable and decrease, where not otherwise stated.

$$m \equiv p - c = -\frac{D(p)}{D'(p)} \equiv \mu(p) \quad (1)$$

where $\epsilon(p)$ is the elasticity of demand. The firm's *market power*, $\mu(\cdot)$, is the ratio of price to elasticity of demand or, in mathematical terms, the inverse hazard rate of demand.

Equation (1) is merely first-order condition. A common condition ensuring its sufficiency for optimization is that demand is log-concave, which is equivalent to market power decreasing in price. This is satisfied by many common demand functions (Weyl, 2008b). Nonetheless it is grossly sufficient as a second-order condition and substantively restrictive. In particular it restricts the pass-through rate $\frac{dp^*}{dc}$ to be less than 1 (Weyl, 2008b). I will therefore generally refer to log-concave demand as “cost-absorbing” and log-convex demand as “cost-amplifying”. In Weyl (2008b) I suggest a simple alternative second-order condition that allows for both cases: the weakest condition for global concavity of the monopolist's problem, namely that $\mu' < 1$, what I there call “mark-up contraction” (MUC). The analog of this relaxation in the RT2003 model will be crucial to allowing the case that violates the conventional wisdom.

Log-concavity, and thus cost absorption, are common assumptions. However, the limited reduced-form empirical evidence on pass-through available suggests that while cost absorption is more common than cost amplification, the second is far from rare (Sidhu, 1971; Sumner, 1981; Besley and Rosen, 1998a,b; Besank et al., 2005). Of course none of this data is directly related to two-sided markets. The best measurements available of pass-through in a two-sided market is from interchange fee interventions by the Australian central bank, which appear to indicate low pass-through rates (Chang et al., 2005). Unfortunately, as (Farrell, 2005) who is skeptical of this conclusion points out, the data set involved is “small, trendy (and) noisy”. Furthermore cost amplification is closely related to fat-tailed distributions of consumer valuations, as is discussed below. Many two-sided markets, such as dating services, seem intuitively to have such large consumer surplus relative to profits. Empirical evidence and theoretical analysis are therefore largely inconclusive as to which case,

cost absorption or cost amplification, is more plausible, though a slight bias in favor of cost absorption seems appropriate.

3.2 Competition

Below I make use of the fact that under symmetrically differentiated Bertrand competition, a merger is equivalent to an increase in market power at every price (on one or more sides of the market). To help clarify the intuition behind this result of RT2003, I briefly review this model in a standard market, which dates to Hotelling (1929) and is treated in a modern way by Tirole (1988).

Consider a market with two symmetrically differentiated substitutes: demand for product 1 when prices for 1 and 2 are respectively p and p' is the same as demand for product 2 when prices are p' and p and both are given by $D(p, p')$. Then goods have common linear cost c of production. There are two natural organizations of this industry to consider: monopoly and duopoly. In the first the two products are owned and sold by a single firm. Assuming a symmetric monopoly optimum, the firm will equate $m = \mu(p)$ where $\mu(\cdot)$ now represents the “total” market power: $\mu(p) = \frac{p}{\epsilon(p)}$ where $\epsilon(p) \equiv \frac{(D_1(p, p) + D_2(p, p))p}{D(p, p)}$ and D_i is the derivative of D with respect to the i th argument. In the second organization, the two firms are in Bertrand competition and the (first-order) equilibrium condition is

$$m = \mu_o(p) \equiv \frac{p}{\epsilon_o(p)} \quad (2)$$

where own-price elasticity of demand $\epsilon_o(p) \equiv -\frac{pD_1(p, p)}{D(p, p)}$. Thus symmetric Bertrand equilibrium occurs at the price that equates mark-up to “own” market power. Clearly because $\epsilon_o > \epsilon$ at every price, as the goods are substitutes, total market power is greater than own market power at every price. Comparing equations (1) and (2), this is clearly the only difference. Furthermore a necessary condition for the stability of the equilibrium is that $\mu'_o < 1$, an analog of MUC. Therefore, the equilibrium price under duopoly is lower than the

monopoly optimal price.

4 The RT2003 model

4.1 Summary

RT2003 is a model of symmetrically differentiated Bertrand duopoly. Just as in a one-sided market, it is equivalent to the monopoly model with lower market power (on both sides of the market), modulo a few technical complications related to allowing market power on each side to depend on prices on both sides. I therefore discuss in the main text the simpler monopoly version of the model and leave to proofs and appendices below the minor complications related to the duopoly model.

Here again I exposit the model in the context of payment cards. Every consumer is endowed with a credit card and every merchant with a machine that allows them to accept cards. A monopolistic card company charges a per-transaction price p^A to consumers and p^B to merchants. Consumers are randomly paired with a merchant, from whom they want to purchase a good, without knowing whether or not they accept cards. Therefore merchants' only motivation to accept cards is that they (exogenously) value consumers to using cards⁴. A fraction of merchants $D^B(p^B)$, derived from these personal valuations, choose to accept cards. A fixed mass of consumers-purchase pairs is matched to these merchants, of which $D^A(p^A)$ choose to use cards, if they have the opportunity to do so; otherwise, they use cash. The monopolist has per-transaction costs c and thus earns profits $(p^A + p^B - c)D^A(p^A)D^B(p^B)$. By analogy to equation (1), the corresponding first-order condition is

$$p^A + p^B - c = \mu^A(p^A) = \mu^B(p^B) \quad (3)$$

where μ^i are derived from demand just as in the standard, one-sided case. Note that here

⁴Rochet and Tirole (2007) relaxes this assumption, allowing merchants to accept cards in order to attract customers and essentially arrives at the same model, but with different normative interpretations.

the definition of the *price level* (RT2006) is entirely clear: $\bar{p} \equiv p^A + p^B$. Furthermore, the *price balance* (as a function of the price level⁵) can be defined simply enough as $(p^A(\bar{p}), \bar{p} - p^A(\bar{p}))$, where $p^A(\bar{p})$ is the optimal or equilibrium price to side A given a particular price level, here defined as solving $\mu^A(p^A) = \mu^B(\bar{p} - p^A)$.

4.2 Analytic frame

A simple way to look at this model is to note that it is exactly⁶ that monopoly pricing problem with two goods where the only cross-effects between the goods is that the price of each good acts as a full cross-subsidy in the production of the other good. That is, every dollar collected from the merchants is equivalent to reducing by one dollar the cost of serving the card-holders⁷. Because $p^A - (c - p^B) = p^B - (c - p^A)$, the mark-up over cost net of this cross-subsidy is by definition the same on the two sides of the market. Therefore given that market power determines the optimal mark-up on one side of the market, the monopolist's equation of market power on the two sides can be seen as a direct consequence of the fact that cross-subsidies are the only cross effects⁸. This way of understanding the RT2003 model forms the basis of the intuitions behind my results below.

RT2003 assumed demand was log-concave on both sides of the market (market power decreasing) to ensure the global concavity of the monopolist's problem. However, this is not necessary for global concavity. Instead, all that is needed is that $\rho_A \rho_B < 1$, a condition I call *cross-subsidy contraction* (CSC).

⁵RT2003 show that the monopolist's optimal price balance for any price level where she earns profits is given by $\mu^A = \mu^B$, not merely her optimal balance at the optimal price level.

⁶To see this note that for this to be the case it must be that optimizing on side A is equivalent to maximizing $(p^A + p^B - c)D^A(p^A)$ and maximizing on side B is equivalent to maximizing $(p^A + p^B - c)D^B(p^B)$. Clearly for both of these to be true any term involving opposite side prices, other than those entering the mark-up, must be multiplicative, yielding the multiplicative form.

⁷The interpretation developed here is closely related to the Lerner (1934) formulae of Rochet and Tirole (2006) who show that each side of the market is priced-to according to a classical Lerner formula, where the cost on each side is off-set by the subsidy from the other side of the market.

⁸As discussed in the proofs below, in the duopoly case when market power may depend on prices on both sides of the market there are also cross-effects on the optimal mark-up on each side, which causes the effective pass-through rate to differ from the pure, demand-determined pass-through rate. However the analysis is otherwise very similar.

Proposition 1. *If demands on the two-sides exhibit CSC and each satisfy MUC, except on a set of measure 0, then equation (3) is sufficient for the monopolist's optimization. If the demands do not jointly satisfy CSC then there exists a pair of prices at which the monopolist's problem is not strictly concave.*

Proof. See Appendix A. □

To see this intuitively, imagine a two-sided markets monopolist who is currently charging prices (p_0^A, p_0^B) . Imagine that p_0^B is her optimal price given p_0^A but p_0^A is slightly too low of a price given p_0^B . To move to the optimum, the monopolist will have to adjust both prices, but we can view her re-optimization sequentially: first she raises p^A to p_1^A to myopically optimize given p_0^B , then she myopically re-optimizes p^B to respond to the rise in p^A to p_1^A . Let $\Delta \equiv p_1^A - p_0^A$ be small. To re-optimize B -side prices the firm simply passes through $\rho^B \Delta$, a fraction ρ^B of the cross-subsidy provided by the increase in side A prices. Re-optimizing on side A the firm must pass-through ρ^A of the tax from the decrease in p^B to consumers, leading them to further increase p^A by $\rho^A \rho^B \Delta$. This process then continues. If $\rho^A \rho^B \geq 1$ clearly this process leads to an infinite increase in p^A and an infinite decrease in p^B . On the other hand if $\rho^A \rho^B < 1$, this process converges, ensuring a first-order solution. I therefore call this condition *cross-subsidy contraction* (CSC).

Demand systems obeying bi-log-concavity ($\rho^A, \rho^B < 1$) are clearly a subset of those obeying CSC. However, CSC also allows for the possibility that $\rho^I > 1 > \rho^J$ for $I = A$ or B . A crucial assumption⁹ I use below is that whichever case holds does so globally. In Weyl (2008b) I show that this assumption holds for nearly all commonly used demand functions and statistical distributions.

Assumption 1. *For both I either $\rho^I(p) < 1$ or $\rho^I(p) > 1$ for all p .*

⁹This assumption is more important in some results than others. For example for local (over-)identification this assumption is not necessary at all, only the second-order conditions are. For the results on competition, the assumption needs only hold over the relevant range of prices and weaker assumptions than this may even be possible (Weyl, 2008b). For the normative results I use the force of the assumption over a wider range of prices, but it only need hold on average over that range.

5 Positive Analysis

A simple way to approach the RT2003 model is to linearize about its first-order conditions. This clearly provides a reasonable approach to the effects of cost shifts. Implicitly differentiating equation (3) yields

$$\frac{\partial p^{A*}}{\partial c} + \frac{\partial p^{B*}}{\partial c} - 1 = \mu^{A'} \frac{\partial p^{A*}}{\partial c} = \mu^{B'} \frac{\partial p^{B*}}{\partial c}$$

or

$$\frac{\partial p^{I*}}{\partial c} = \frac{\rho^I(1 - \rho^J)}{1 - \rho^I \rho^J} \quad (4)$$

$$\bar{\rho} \equiv \frac{\partial \bar{p}^*}{\partial c} = 2 + \frac{\rho^A + \rho^B - 2}{1 - \rho^A \rho^B} \quad (5)$$

The intuition behind these formulae is simple. All increases in cost not passed through to side A of the market are passed through to side B at its pass-through rate, except that this whole process is amplified by the standard inverse stability factor, here $\frac{1}{1 - \rho^A \rho^B}$. The “natural” total pass-through is 2 (full pass-through to each side); anything above or below this is amplified by the same inverse stability factor. The most useful thing about these formulae is that they help establish a valuable proposition. Namely that (under monopoly) either both prices are co-monotonic with cost and the demand system is *total cost absorbing* in the sense that $\bar{\rho} < 1$ or one price is anti-co-monotonic with cost and the demand system is *total cost amplifying* in the sense that $\bar{\rho} > 1$. This means that the monopoly model is locally over-identified. Furthermore, ρ^A and ρ^B can obviously be recovered locally with quantitative precision and without parametric assumptions.

Proposition 2. *Under monopoly assuming (strict) CSC and MUC either*

1. $\frac{\partial p^{I*}}{\partial c} > 0$ for both I , the demand system is locally cost-absorbing and both demands are locally cost-absorbing.

2. $\frac{\partial p^{I^*}}{\partial c} = 0$, $\frac{\partial p^{J^*}}{\partial c} = 1$ and $\rho^J = 1$ for some assignment of I, J .

3. Or $\frac{\partial p^{I^*}}{\partial c} < 0 < \frac{\partial p^{J^*}}{\partial c}$ for some assignment of I, J , the demand system is locally cost-amplifying and side I and J demand are respectively locally cost-absorbing and locally cost-amplifying.

Proof. First note that it cannot be that both $\frac{\partial p^{A^*}}{\partial c}$ and $\frac{\partial p^{B^*}}{\partial c}$ are weakly negative as the sign of the first is (by CSC) the same as $\rho^A(1 - \rho^B)$ and the second $\rho^B(1 - \rho^A)$ which have the same sign as $1 - \rho^B$ and $1 - \rho^A$ respectively by MUC. Clearly if both of these are weakly negative, CSC is violated.

If $\frac{\partial p^{A^*}}{\partial c}$ and $\frac{\partial p^{B^*}}{\partial c}$ are both strictly positive, then clearly $\rho^A, \rho^B < 1$. Note that the sign of $\bar{\rho} - 1$

$$\text{sign}\left(1 + \frac{\rho^A + \rho^B - 2}{1 - \rho^A \rho^B}\right) = \text{sign}(\rho^A + \rho^B - \rho^A \rho^B - 1) = -\text{sign}([\rho^A - 1][\rho^B - 1])$$

which is clearly negative if $\rho^A, \rho^B < 1$. If $\frac{\partial p^{I^*}}{\partial c} = 0$ then clearly $\rho^J = 1$ and $\bar{\rho} = 1$. If $\frac{\partial p^{I^*}}{\partial c} < 0$ clearly $\rho^B > 1 > \rho^A$ and $\bar{\rho} > 1$. \square

Intuitively prices will fall to side I when costs rise if side J more than fully absorbs the cost rise. Furthermore, at the first round, anything not passed-through to one side is passed-through to the other so that if $\rho^I < 1$ the first-round total pass-through is $\rho^I + (1 - \rho^I)\rho^J$, which is clearly a convex combination of 1 and ρ^J and will therefore lie on the same side of 1 as ρ^J . Thus the same parameters driving total cost amplification versus absorption determine whether prices are co- or anti-co-monotonic in cost.

The above formulae are complicated slightly when considering the comparative statics under Bertrand duopoly, as market power depends on prices on both side of the market. Nonetheless, the following result still holds which, while insufficient for testing the models, does allow identification of the effects of mergers from that Bertrand equilibrium. Note that

the cost shocks here must be common to the two firms, given the assumption of symmetry.

Proposition 3. *Consider the RT2003 model of competition, with equilibrium conditions given by*

$$p^A + p^B - c = \mu_o^A(p^A, p^B) = \mu_o^B(p^B, p^A) \quad (6)$$

and assume the equivalent of MUC and CSC $\mu_{o,1}^A, \mu_{o,1}^B < 1$ and $\tilde{\rho}^A \tilde{\rho}^B < 1$ where $\tilde{\rho}^I \equiv \rho^I(1 - \mu_{o,2}^I)$ where $\mu_{o,i}^I$ denotes the derivative of μ_o^I with respect to its i th argument. Then there are again three possibilities:

1. $\frac{\partial p^{I*}}{\partial c} > 0$ and $\tilde{\rho}^I < 1$ for both I .
2. $\frac{\partial p^{I*}}{\partial c} = 0 < \frac{\partial p^{J*}}{\partial c}$ and $\tilde{\rho}^J = 1$
3. $\frac{\partial p^{I*}}{\partial c} < 0 < \frac{\partial p^{J*}}{\partial c}$ and $\tilde{\rho}^I < 1 < \tilde{\rho}^J$ for some assignment of I, J .

Proof. See Appendix B □

The effects of competition are not local, leading to some complexities treated in the appendix. However, because the introduction of competition is equivalent to lowering market power, to gain a heuristic sense for the effects of competition, we can consider local shifts in market power, starting from the monopoly model. First consider an exogenous fall in market power on side I of a (differentially) small size Δ . This corresponds to the introduction of competitive pressure on only one side of the market, a sort of “completely unbalanced” competition which can either represent de-cartelization of one side of the market, leaving pricing on the other cartelized, competition in the RT2003 for the correct choice of competition parameters¹⁰ or competition with a firm producing a product that acts as a substitute for the platform for consumers on one side of the market.

¹⁰Competition in the RT2003 model for “buyers” (consumers) is driven by marginal substitutability, while competition for “sellers” (merchants) is driven by a sort of average substitutability called the degree of “multihoming” (how many consumers who when able to use either card pay with a card will, only given the option to use their less preferred card, will still choose to use a card over cash). Completely unbalanced competition favoring buyers is therefore difficult to generate, except in the limit: it requires that while consumers are willing to substitute between the cards on the margin, no infra-marginal consumers are

$$\frac{\partial p^I}{\partial \Delta} = -\frac{\rho^I}{1 - \rho^I \rho^J} \quad (7)$$

$$\frac{\partial p^J}{\partial \Delta} = \frac{\rho^I \rho^J}{1 - \rho^I \rho^J} \quad (8)$$

$$\frac{\partial \bar{p}}{\partial \Delta} = \frac{\rho^I(\rho^J - 1)}{1 - \rho^I \rho^J} \quad (9)$$

There are a few things worth noting.

1. Unsurprisingly, unbalanced competition reduces prices on the side of the market where it takes place and raises them on the opposite side of the market, what RT2003 call the “topsy-turvy” effect and Lerner and Tirole (2008) call, as I do below, the “seesaw” effect.
2. Unbalanced competition on side I will reduce the price level if and only if side J absorbs, rather than magnifies, the cross-tax on side J resulting from the fall in side I prices. Thus the counter-intuitive possibility that competition *increases* the price level will occur as a result of “weird” (cost-amplifying) demand on the side *opposite* where the competition takes place; unbalanced competition on the cost-amplifying side will reduce the price level.
3. Price controls on one side of the market have exactly the same effect as unbalanced competition favoring that side: they will tend to reduce prices on that side and raise them on the other, possibly more than compensating the reduction if the opposite side has cost-amplifying demand.
4. A simple calculation shows the intuitive result that a very balanced form of competition (including market power falling multiplicatively or additively by the same amount on

willing to use their less preferred card. On the other hand completely unbalanced competition favoring sellers is easy to generate, as it simply requires perfect differentiation on the margin, while consumers are all willing to use the other card if push comes to shove. Explicit constructions of these cases are available on request.

the two sides of the market) of size Δ will have the same effect on prices as a fall in cost. It will therefore always reduce the price level and will reduce prices on both sides of the market under total cost absorption, while increasing prices on the cost-absorbing side under total cost amplification. This sort of competition is also equivalent to a price control on the sum of the two prices.

5. Because (exogenous) cost variations identify whether demands on each side of the market are cost-amplifying or cost-absorbing, these combined with Assumption (1) above identify the sign of the effects of competition¹¹.
6. Perhaps less obviously, all of these results apply starting from competition (i.e. an “unbalanced” merger or cartelization has the same sort of effect as unbalanced competition and its effects can be identified through cost variations in the same way starting from competition).

These are expressed in the following two propositions, which maintain

Assumption 2. *MUC, CSC and their Bertrand analogs hold before and after all shifts in industrial organization. Also assume a slightly stronger cross form of CSC, name that in any shift between the duopoly and monopolistic cases $\rho^I \tilde{\rho}^J < 1$ for all price pairs and assignments of I, J .*

Proposition 4. *Maintaining Assumption 1 and 2, if completely unbalanced competition (market power is strictly lower under duopoly only on one side) or a binding price control is introduced on side I then p^I falls, p^J rises and the price level will fall if $\rho^J < 1$ and rise if $\rho^J > 1$. Perfectly balanced competition (duopoly price balance is monopoly optimal) or a cap on the price level, allowing the monopolist to optimize balance, will always reduce the price level and will reduce (increase) prices on any side of the market I such that $\rho^{-I} < (>)1$.*

¹¹And, locally, even its magnitude if cross-elasticities are available, in an extension of Froeb et al. (2005)’s argument that pass-through rates combined with diversion rates (cross-elasticities) are sufficient to predict, locally, the effects of mergers.

Proof. See Appendix C. □

Proposition 5. *Maintaining now the analog of Assumption (1), replacing ρ^I with $\tilde{\rho}^I$ and Assumption 4, a completely unbalanced merger or collusion (market power rises on only one side of the market) or a binding price floor on side I then p^I rises, p^J falls and the price level will fall if $\tilde{\rho}^J < 1$ and rise if $\tilde{\rho}^J > 1$. Perfectly balanced collusion (monopoly optimal price balance equates own market power on the two sides) or a floor on the price level will always raise the price level and will increase prices on any side of the market I such that $\tilde{\rho}^{-I} < 1$.*

Proof. See Appendix C. □

5.1 Conventional wisdom

Insert Figure 1 here

Under total cost absorption, prices behave like a seesaw which, rather than sitting on the ground, is suspended by its axel from a rubber band as pictured in Figure 1. Competitive pressure or price controls are represented by the weight shown. When applied to one side of the market only (one end of the seesaw) they will raise prices on the opposite side and reduce them on their side; because of the rubber band, they will also reduce the overall price level. On the other hand when applied evenly across the two sides of the market, price controls or competition will reduce both prices, maintaining the balance while stretching the rubber band. This confirms the conventional wisdom: price controls and competition can only be relied upon to reduce the price level; their effect on the balance of prices depends on the particulars of competitive dynamics, the type of price control etc. Furthermore, competition need not reduce prices on both sides of the market: while the immediate effect of competition is to reduce prices, a reduction of prices on one side of the market acts as a tax on the other side which can overwhelm the direct effects of competition. In fact if competition takes place (elasticity rises), or a price control is instituted, only on one side of the market that side will achieve lower prices while the other side's price rises. Only "balanced" interventions, such

as subsidies, controls on the price level and competition when it is balanced can be relied on to reduce prices on both sides.

In addition to confirming the conventional wisdom this helps resolve some of the disagreement in the literature. Chakravorti and Roson (2006)'s claim that competition reduces prices on both sides of the market neglects the (indirect) effect of competition on one side on prices on the other side through the incentives of the firm. Bolt and Tieman (2005), Roson (2005) and DeGrauwe et al. (2007)'s argument that a rise in elasticity on one side actually raises prices on that side and lowers it on the other is due to a mistaken interpretation of the price balance condition $\frac{p^A}{\epsilon^A(p^A)} = \frac{p^B}{\epsilon^B(p^B)}$. It is true that, at equilibrium, the side with the higher price also has a higher elasticity. However the effect of a shift in elasticity *as a function* depends on whether the $\epsilon'(p) < 1$ or $\epsilon'(p) > 1$. Under cost absorption, in contrast to the constant elasticity case from which this mistaken intuition likely arises, market power decreases with price, so an increase in elasticity (as a function) lowers, not raises, the relative price on that side of the market.

5.2 Conventional wisdom challenged

Insert Figure 2 here.

Under total cost amplification, the physical analogy changes a bit. Now the rubber band, rather than attaching to the ceiling, passes over a pulley and attaches its other end to the cost-absorbing side of the market, as pictured in Figure 2. Now pressure applied to the cost-absorbing side alone will actually raise the price level by pulling the seesaw back over the pulley, while pressure applied to the cost-amplifying side continues to lower the price level (but of course, still raises prices on the cost-absorbing side). Evenly applied pressure reduces the price level, but that pulls the prices on the cost-absorbing side, attached to the rubber band, back over the pulley, raising its price.

This case is entirely contrary to the conventional wisdom. Competition cannot be relied on to reduce the price level, in fact it may increase it. Competition that is balanced, and

other balanced interventions, no longer have an ambiguous effect on the balance of prices, nor can they be relied on to reduce both prices; rather they consistently raise prices to one side and lower them to the other. Clearly this case has very different policy implications than those of the conventional wisdom, though I defer this discussion to my normative analysis below. Propositions 2 and 3 avoid the ambiguity this creates for policy makers, as they allow, along with the fairly weak Assumption 1, a non-parametric identification of which case prevails, as well as, in the monopoly case, a test of the model.

6 Normative Analysis

By the assumption of multiplicative demand, the value each side gains from participating in the system is proportional to the number of consumers that participate on the other side of the market. This is the basic two-sided externality. Therefore the “gross” surplus (RT2003) on each side $V^I(p^I) \equiv \frac{\int_{p^I}^{\infty} D^I(p) dp}{D^I(p^I)}$ must be multiplied by the opposite side demand to generate a formula for consumer surplus on side I $V^I(p^I)D^J(p^J)$. Therefore total consumer surplus (with even weightings) is given by

$$V^A(p^A)D^B(p^B) + V^B(p^B)D^A(p^A)$$

and social surplus adds to this the monopolist’s profits $(p^A + p^B - c)D^A(p^A)D^B(p^B)$, assuming, again even weighting on the firm’s profits. Perhaps surprisingly, RT2003 show that under Bertrand duopoly these formulas continue to hold as, at equilibrium, all consumers use their most preferred card. However, most of the results below take advantage of other elements of the monopoly setting and are thus special to it.

A main contention of the conventional wisdom is that the price balance is unlikely to be very distorted, or at least that it is likely to be difficult to improve it through intervention. To make sense of this, a basic question we might ask is whether it is possible, starting at monopoly optimal price balance, given any price level \bar{p} , to improve the average welfare of

consumers on *both* sides of the market by changing the price balance. While this would not indicate a true Pareto inefficiency as it lumps together consumers on the two sides of the market, it might be considered a quasi-Pareto inefficiency and a clear distortion of the price balance. Whether this is possible turns on whether the average consumer on one side of the market would ever like to have a tax levied against him to finance a reduction in prices to the other group of consumers. The average consumer on side I would like such a tax to be levied if and only if he gains more from a consumer participating on side J than the monopolist does. Because the monopolist's profits per consumer are given by her market power and the average consumer's benefits are given by her average surplus, consumers on side I will want their prices to if $\bar{V}^I(p^I) > \mu^I(p^I)$; if $\mu^I > \bar{V}^I$ the average consumer on side I of the market would like to tax consumers on the opposite side to subsidize themselves, as is more intuitive. The conditions in which these two cases occur are, surprisingly, also closely related to pass-through as shown by Fabinger and Weyl (2008).

Proposition 6. *Given Assumption 1, if the demand system is total cost absorbing then it is never the case that (the average consumer on) both sides of the market can be made better off by a transfer from one side to the other¹². Conversely if the demand system is total cost amplifying and demand on the cost-amplifying side is strictly positive at all prices, a small transfer from the cost-amplifying side to the cost-absorbing side will always make (the average consumer of) both better off.*

Proof. The derivative of I side welfare has sign

$$\bar{V}^I - \mu^J \tag{10}$$

At the monopoly's optimal price balance $\mu^I = \mu^J$, proving that a transfer from I to J benefits I (starting at monopoly optimal price balance) if and only if $\mu^I < \bar{V}^I$. But Fabinger and Weyl (2008) show that for any p $\bar{V}(p) = \mu(p)\bar{\rho}(p)$ where $\bar{\rho}(p)$ is a weighted average,

¹²Bedre and Calvano (2008) independently arrived at a result closely related to the first half of this proposition after an earlier version of this paper was circulated, but before this draft.

the exact formula for which is omitted here, of ρ at prices above p . I will refer to this as *average pass-through*. Thus under assumption 1 cost absorption implies $\mu^I < \bar{V}^I$ and cost amplification that $\mu^I > \bar{V}^I$.

□

The average consumer on one side of the market may benefit from being taxed to subsidize consumers on the other side of the market if their average value of interacting with consumers on the other side is high enough. Such transfers do not take place through separate Coasian bargaining by the assumption inherent to the RT2003 model that such Coasian bargaining is ruled out, though perhaps this becomes less plausible in the case when such strong incentives exist for it to take place. Even when such quasi-Pareto improving transfers are possible it may not be in the interest of the monopolist to carry them out, as she only internalizes the *marginal* surplus of consumers, not their *average* surplus because she cannot perfectly price discriminate¹³. When this infra-marginal surplus is large relative to market power, the monopolist becomes nearly indifferent between chasing infra-marginal consumer surplus and continuing to serve a larger market. This indifference leads her to respond sharply to changes in costs which break this indifference, leading to high pass-through rates. Therefore the basic normative comparative statics of two-sided markets are also driven by pass-through.

A weaker notion of distortion of the price balance considered by RT2003 is that an improvement to consumer, as they consider, or social welfare is possible starting at monopoly optimal prices by a transfer. RT2003 show that, surprisingly, if demands on both sides of the market are linear, the monopolist actually chooses the consumer and socially optimal price balance for any given price level. Of course, as RT2003 recognize, there should be no general supposition that the socially optimal balance of prices. As the following proposition shows, this will occur only when the ratio of consumer to producer surplus (which is determined,

¹³This suggests that there may be additional benefits to price discrimination in two-sided market. For example they might even benefit the discriminated *against* sub-group of consumers if this group has sufficiently high average surplus. Thus the high private incentive that high pass-through rates create for private incentives may be matched by a social incentive in two-sided markets.

as discussed above, by the average pass-through) is the same on both sides of the market.

Proposition 7. *Assume that the natural second-order condition, discussed in Appendix D, are satisfied for the social and consumer optimal price balance problems. Then the monopolist chooses socially optimal price balance if and only if $\bar{p}^A = \bar{p}^B$ at the monopolist's optimal prices.*

Corollary 1. *Under Assumption 1 the monopolist can choose the socially optimal price balance only under total cost absorption.*

Proof. See Appendix D. □

This gives a second sense in which the price balance is more likely to be distorted under total cost amplification than total cost absorption: under total cost amplification the price balance is always bounded away from the optimum while it may achieve this optimum under total cost absorption¹⁴. Of course knowing that the price balance is not optimal is not enough for a social planner to improve on the monopoly's price balance. Instead, she must know which direction it would be beneficial to move the balance in. The following proposition provides a third reason why it may be more difficult to improve on the monopoly optimal price balance under total cost absorption than amplification: it is more difficult to identify the right direction for the price balance to move under total cost absorption.

Proposition 8. *Starting at monopoly optimal price balance given any price level weakly above cost so long as average pass-through rates differ on the two sides of the market, social and consumer surplus can be increased by a small transfer from the side of the market with high average pass-through to that with low. If the second-order conditions discussed in Proposition 7 are obeyed, then the socially and consumer, respectively, optimal price balance can be achieved by a sufficiently large transfer of this sort.*

¹⁴This result, and the following one, should be taken with some caution though; while they emphasize the distinction between total cost absorption and amplification as these are stark, given other assumptions that divided pass-through rates on one side of the market from those on the other globally, similar results could likely be obtained. The somewhat arbitrary (for these two results only) cutoff of total cost absorption versus amplification is still, I think, analytically useful, conveying the spirit of welfare economics in two-sided markets and bringing them into a unified framework with the other normative and positive results.

Corollary 2. *Under assumption 1, in the case of total cost absorption, knowing which direction to move the price balance requires knowledge of average pass-through rates on the two sides, while under total cost amplification it is always optimal to raise prices to the (locally) cost-amplifying side while lowering them to the (locally) cost-absorbing side.*

Proof. The derivative of consumer welfare with respect to a transfer from A to B is from above proportional to

$$\bar{p}^A \mu^{A^2} - \bar{p}^B \mu^{B^2} \propto \bar{p}^A - \bar{p}^B$$

as $\mu^A = \mu^B$ at monopoly optimal prices. Note that there is obviously no first-order effect on monopoly profits of a transfer we start at monopoly optimal prices. Therefore this result holds for social surplus as well. The global versions hold because as shown in Appendix D the relevant conditions are the second-order conditions of the socially and consumer optimal balance problems. \square

At monopoly optimal prices, transfers have no first-order effect on the volume of transactions, as a monopolist has maximized these given the price level. Their only effect on consumer welfare therefore is to increase the number of consumers *opposite* on the transferred-to side of the market and reduce the number of consumers on the transferred-from side of the market. Given that the high average pass-through group of consumers value partners more, it is efficient to make the transfer from the low to the high average pass-through side.

Of course this heuristic works only if one starts at monopoly optimal prices. A monopolist who knows a policy maker is likely to impose regulations on price balance will have an incentive to manipulate her information. Nonetheless, this provides a useful benchmark for thinking about how to improve the balance of prices. It also suggests a problematic implication of the total cost amplification case. When the price level falls but the monopolist maintains her optimal price balance, under cost amplification prices fall to the cost amplifying side of the market and rise to the cost absorbing side. While the price level falling is beneficial,

this moves the price balance in exactly the wrong direction. Thus the price level or costs falling may not even benefit consumer or social welfare under total cost amplification. This is in sharp contrast to the case of total cost absorption, when even under monopoly governance of the price balance, the socially optimal price level is always strictly below cost because of the infra-marginal externalities across the two sides of the market¹⁵. Clearly under total cost absorption both sides of the market always benefit when the price level falls as prices fall to both groups of consumers, decreasing the price and increasing the external benefits of other-side membership to both.

Proposition 9. *Given Assumption 1, under total cost absorption (for a monopoly or $\tilde{\rho}^A, \tilde{\rho}^B < 1$ for duopoly) the socially optimal price level given that price balance is determined by $\mu^A = \mu^B$ (or $\mu_o^A = \mu_o^B$ in the duopoly case) is strictly below cost. Conversely under total cost amplification the socially optimal price level may be above cost and consumers can even benefit from an increase in the price level above its monopoly (or duopoly) level.*

Corollary 3. *Under total cost amplification, but not under total cost absorption, a rise in firm costs may benefit (average) consumers and society .*

Proof. See Appendix E. □

Despite the benefits of price level reductions in the total cost absorption case, even there these can be harmful if brought about in an unbalanced way by a price control on only one side of the market or unbalanced competition, as this may distort the price balance in the wrong direction.

Proposition 10. *A price control on only one side of the market (under either monopoly or duopoly) or the introduction of completely unbalanced competition may harm consumer (and*

¹⁵While it may seem intuitive that prices should be below their 0-market power level in two-sided markets, this result is driven by consumers being heterogeneous in their usage valuations (Weyl, 2008a). This leads monopolists to fail to internalize their full surplus. By contrast in the Armstrong (2006) model, socially optimal prices are exactly those chosen by a monopolist with no market power (were she to chose to operate). I discuss the relationship among the monopoly, Ramsey and Lindahl problems in the RT2003 model in greater detail in Weyl (Forthcoming).

therefore social) welfare under either total cost absorption or total cost amplification while maintaining Assumption 1.

Proof. See Appendix F. □

6.1 Conventional wisdom

Under total cost absorption these results again conform closely to the conventional wisdom. In several formal senses the price level is more distorted than the price balance and balanced interventions, aimed at reducing the price level rather than shifting its balance, are more reliably beneficial than those that seek to adjust the price balance. Furthermore these results show that the claim by Economides and Tåg (2007) that price controls on one side of the market improve welfare is special to the linear demand system they employ. The policy implications of this conventional wisdom have been extensively discussed elsewhere, but I will briefly summarize them here. Reducing the price level rather than changing its balance should be the aim of policy. The price level rather than individual prices is the appropriate means of measuring the competitiveness of a market for purposes of market definition, merger analysis, etc. Subsidies may be desirable as, in any networked market, positive externalities exist. Balanced interventions and competition have little systematic effect on the balance prices, their main effect being to reduce the price level. This case is largely like standard industrial organization, so long as a focus on individual prices is replaced by the price level.

6.2 Conventional wisdom challenged

Under total cost amplification the conventional wisdom is largely turned on its head. The price balance may well be more distorted than its level; in fact, the optimal price level may be above cost given monopoly governance of the balance. The policy implications of these results, combined with the positive results above, conflict deeply with those of the conventional wisdom. Competition may well raise the price level and the price level cannot be

used to gauge the competitiveness of an industry. Balanced interventions aimed at reducing the price level do not have balanced effects, systematically tend to shift the price balance in favor of the cost-amplifying side of the market and may be harmful. There may be a realistic prospect of identifying and implementing an improvement in the price balance, something which consumers on both sides of the market may benefit from. This case is very dissimilar from a standard market and may call for more heavy-handed regulation.

7 Conclusion

This paper comprehensively analyzes the positive and normative comparative statics of policy in the RT2003 model of two-sided markets. It shows that while the traditional conventional wisdom about policy based on this model holds in one case, it is entirely over-turned in another not considered previously. Luckily, the distinction between these two cases can be identified, and the model tested under monopoly, given exogenous cost variations. It also helps resolve a number of disputes within the literature.

While I briefly discuss the policy implications of these different cases, making the theoretical results here directly useful for applied policy analysis is an important direction for future research, particularly in areas such as market definition, collusion, regulation (such as net-neutrality) and other pressing policy issues in two-sided markets. There are also a number of more theoretical directions for future research that are promising. Given the results here it should be fairly simple to extend the comparative statics of the RT2003 model to the case of N-sided markets, though the justification of their multiplicative demand form from a primitive model, particularly in the case of competition, might be challenging. Extensions to more than two competing firms and asymmetric Bertrand competition are important for enriching the realism of the model. Finally, pass-through may be useful as a means for analyzing more general models of two-sided markets; this is an avenue I am pursuing in work in progress (Weyl, 2008a).

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Appendix

A Proof of Proposition 1

The monopolist’s objective function is $(p^A + p^B - c)D^A(p^A)D^B(p^B)$. The Hessian for this problem when the first-order conditions from equation 3 are satisfied is

$$\begin{bmatrix} 2D^A D^{B'} + D^{A''} D^B (\bar{p} - c) & D^{A'} D^B + D^A D^{B'} + D^{A'} D^{B'} \\ D^{A'} D^B + D^A D^{B'} + D^{A'} D^{B'} & 2D^{A'} D^B + D^A D^{B''} (\bar{p} - c) \end{bmatrix} =$$

$$D^{A'} D^{B'} \mu \begin{bmatrix} -\frac{1}{\rho^A} & -1 \\ -1 & -\frac{1}{\rho^B} \end{bmatrix}$$

where $\mu \equiv \mu^A = \mu^B$ as the first-order conditions are satisfied. This is negative definite if and only if

$$\rho^A, \rho^B > 0 \tag{11}$$

and

$$\rho^I > 1$$

as $D^{A'} D^{B'} > 0$ as both demands are assumed decreasing. Thus CSC is necessary and sufficient for the inequality (11) and MUC on both sides of the market clearly implies $\rho^A > 0$. If CSC is violated and therefore so is the inequality (11) then clearly the expression is not strictly quasi-concave, except possible in the case when these are satisfied with weak inequality on a set of measure 0.

B Proof of Proposition 3

A proof that the equivalent of CSC and MUC in this context are necessary stability conditions is available on request and intuitive from what follows, but omitted here for the sake of brevity. Using the same strategy as in the proof of Proposition 2

$$\frac{\partial p^I}{\partial c} + \frac{\partial p^J}{\partial c} - 1 = \mu_1^I \frac{\partial p^I}{\partial c} + \mu_2^I \frac{\partial p^J}{\partial c}$$

$$\frac{\partial p^I}{\partial c} = \rho^I \left(1 - [1 - \mu_2^I] \frac{\partial p^J}{\partial c} \right)$$

where $\rho^I \equiv \frac{1}{1-\mu_1^I}$. Substituting in

$$\frac{\partial p^I}{\partial c} = \rho^I \left(1 - [1 - \tilde{\rho}^J] \left[1 - (1 - \mu_2^J) \frac{\partial p^I}{\partial c} \right] \right)$$

where $\tilde{\rho}^J \equiv (1 - \mu_2^J)\rho^J$. Simplifying

$$\frac{\partial p^I}{\partial c} = \frac{\rho^I(1 - \tilde{\rho}^J)}{1 - \tilde{\rho}^I \tilde{\rho}^J} \quad (12)$$

By MUC $\rho^A, \rho^B > 0$ so the result is established from this exactly as in the proof of Proposition 2.

C Proof of Proposition 4 and 5

First consider the effect of unbalanced competition. WLOG let the competition take place on side A . Let the functions $p^A(t), p^B(t) : [0, 1] \mapsto \mathbb{R}$ be defined by

$$p^A(t) + p^B(t) - c = (1 - t)\mu^A(p^A[t]) + t\mu_o^A(p^A[t], p^B[t]) = \mu^B(p^B[t]) \quad (13)$$

Note that by definition, $(p^A[0], p^B[0])$ is the monopoly optimal price pair and $(p^A[1], p^B[1])$ is the Bertrand equilibrium price pair. Let $\Delta(t) \equiv \mu_o^A(p^A[t], p^B[t]) - \mu^A(p^A[t]), \tilde{\rho}^B(t) \equiv$

$\rho^B(p^B[t]) \left(1 - t\mu_{o,2}^B[p^A(t), p^B(t)]\right), \rho^A(t) \equiv \frac{1}{1 - (1-t)\mu^{A'}(p^A[t]) - t\mu_{o,1}^A(p^A[t], p^B[t])}$. Note that by definition $\Delta > 0$,

$$0 < \min \left\{ \frac{1}{1 - \mu^{A'}}, \frac{1}{1 - \mu_{o,1}^A} \right\} < \rho^A < \max \left\{ \frac{1}{1 - \mu^{A'}}, \frac{1}{1 - \mu_{o,1}^A} \right\}$$

and

$$\tilde{\rho}^B < \max \{ \rho^B, \rho^B(1 - \mu_{o,2}^B) \}$$

therefore clearly $\rho^A \tilde{\rho}^B < 1$. Differentiating the system in equation(s) (13) yields

$$p^{A'} + p^{B'} = -\Delta + (1-t)\mu^{A'} p^{A'} + t(\mu_{o,1}^A p^{A'} + \mu_{o,2}^A p^{B'}) = \mu^{B'} p^{B'}$$

so

$$p^{B'} = -\rho^B p^{A'}$$

and

$$p^{A'}(1 - \rho^B) = -\Delta + ([1-t]\mu^{A'} + t\mu_{o,1}^A)p^{A'} - \mu_{o,2}^A \rho^B p^{A'}$$

this yields

$$p^{A'} = -\frac{\rho^A \Delta}{1 - \rho^A \tilde{\rho}^B} \tag{14}$$

$$p^{B'} = \frac{\rho^A \rho^B \Delta}{1 - \rho^A \tilde{\rho}^B} \tag{15}$$

$$\bar{p}' = \frac{\rho^A(\rho^B - 1)\Delta}{1 - \rho^A \tilde{\rho}^B} \tag{16}$$

where $\bar{p} \equiv p^A + p^B$. These are all obviously signed in the direction posited in the proof and

therefore so is $p^I(1) - p^I(0)$ and $\bar{p}(1) - \bar{p}(0)$. The proof of the unbalanced merger result is exactly analogous except that $\tilde{\rho}^A$ must also be defined (analogously to $\tilde{\rho}^B$) and replaces ρ^A in the denominator of the above expressions. A more detailed proof is, of course, available on request.

For the balanced competition result, note that if, at the new equilibrium it continues to be the case that $\mu^A = \mu^B$ then, at those prices, $\mu_o^I = \mu^I - \alpha$ for both I and a common α . Thus if we define $p^A(t), p^B(t) : [0, 1] \mapsto \mathbb{R}$ by

$$p^A(t) + p^B(t) - c = \mu^A(p^A[t]) - t\alpha = \mu^B(p^B[t]) - t\alpha \quad (17)$$

clearly by the same argument as before $(p^A[0], p^B[0])$ are the monopoly optimal prices and $(p^A[1], p^B[1])$ are the Bertrand equilibrium prices. However now note that equation (17) can be rewritten as

$$p^A(t) + p^B(t) - c + t\alpha = \mu^A(p^A[t]) = \mu^B(p^B[t])$$

Thus $p^{I'} = \alpha \frac{\partial p^A}{\partial c}$, the sign of which is given by equations (4) and (5), establishing the result. Note that this case also clearly applied to a common multiplicative shift in both market power functions

For price controls on one side of the market, leaving the other unregulated, note that relative to the equilibrium, the price on the regulated side must fall by the price control being binding. However as the other price is not controlled, it must continue to be governed by the equation of mark-up over cross-subsidized cost to market power. Thus it must rise as the other price falls, and by more (less) than the amount of the fall in the other price if pass-through is greater (less) than 1. The opposite argument establishes the Bertrand price floor case.

Finally for the cap on the sum of the two prices, this reduces the price level but maintains the balance condition and thus has the same effect as balanced competition or a reduction

in cost, establishing the result. A price floor starting at competition has the opposite effect. More formal proofs of the two price control results are available on request, but are so similar to those above that I omit them here.

D Proof of Proposition 7

RT2003 show that the consumer welfare optimal price balance is given by

$$\mu^A \bar{V}^A = \mu^B \bar{V}^B$$

or by Fabinger and Weyl (2008)'s formula

$$\bar{\rho}^A = \bar{\rho}^B$$

This clearly will be solved at the same prices solving $\mu^A = \mu^B$ if and only if $\bar{\rho}^A = \bar{\rho}^B$ at those prices. By RT2003's reasoning, the derivative of consumer surplus with respect to a transfer from side A to side B is

$$\mu^A \bar{V}^A - \mu^B \bar{V}^B$$

Therefore a (weak) sufficient second-order condition is

$$\mu^{A'} \bar{V}^A + \mu^A \bar{V}^{A'} + \mu^{B'} \bar{V}^B + \mu^B \bar{V}^{B'} < 0$$

or equivalently by a bit of algebra

$$\sum_{I=A,B} \mu^I \left(2\bar{\rho}^I - \frac{\bar{\rho}^I}{\rho^I} - 1 \right) < 0$$

Following the same logic but adding in the firm's profits yields the social surplus second-order

balance condition:

$$\frac{(\rho^A - 1)(\bar{V}^A + \bar{p} - c)}{\rho^A} + \bar{V}^A - \mu^A + \frac{(\rho^B - 1)(\bar{V}^B + \bar{p} - c)}{\rho^B} + \bar{V}^B - \mu^B < 0$$

These automatically occur under total cost absorption and Assumption 1 so long as the price level is above its Lindahl level (Weyl, Forthcoming).

E Proof of Proposition 9

To show the first half, I need to show that so long as both prices fall when the price level does and so long as the price level is weakly above cost, social welfare will strictly improve with a fall in the price level. This is because either $\rho^A, \rho^B < 1$ and monopoly or under $\tilde{\rho}^A, \tilde{\rho}^B < 1$ and competition, a reduction in the price level lowers both prices. Let $\sigma^I \equiv \frac{dp^I}{d\bar{p}}$ when a given balance condition is maintained. Then taking the derivative of social welfare with respect to a reduction in the price level yields

$$\sigma^A D^{A'}(V^B + \bar{p} - c) + \sigma^B D^{B'}(V^A + \bar{p} - c)$$

which is clearly negative for $\bar{p} \geq c$ as $V^I > 0$ and $D^{I'} < 0 < \sigma^I$ so long as a reduction in the price level lowers prices to both groups. The reason is the positive externalities each group brings to the other and the fact that with prices weakly above cost, the classic monopoly problem indicates that the price level should be reduced.

The derivative above has the same sign, dividing through by $D^{A'} D^{B'}$ as

$$-\sigma^A \bar{V}^B - \sigma^B \bar{V}^A - (\bar{p} - c) \tag{18}$$

as $\sigma^A + \sigma^B = 1$ by definition. Note that, of course, σ^I may be negative on one side of the market. σ^I is given by dividing expression (4) by expression (5):

$$\sigma^I = \frac{\rho^I(1 - \rho^J)}{\rho^I + \rho^J - 2\rho^I\rho^J} \quad (19)$$

The second-order condition associated with the first-order condition in (18)

$$-\frac{\partial\sigma^A}{\partial\bar{p}}\bar{V}^B - \frac{\partial\sigma^B}{\partial\bar{p}}\bar{V}^A - \sigma^A\sigma^B(\bar{\rho}^A + \bar{\rho}^B - 2) < 1 \quad (20)$$

If demand has constant pass-through, this simplifies to

$$-\sigma^A\sigma^B(\rho^A + \rho^B - 2) < 1 \quad (21)$$

On the other hand if demand has constant pass-through, at $\bar{p} = c$ expression (18) simplifies to

$$-\sigma^A\rho^B\mu^B - \sigma^B\rho^A\mu^A$$

Given that $\mu^A = \mu^B$ at the monopoly optimal price balance the sign of this is the same as that of

$$-\rho^A\rho^B(\rho^A - 1) - \rho^A\rho^B(\rho^B - 1) = -\rho^A\rho^B(\rho^A + \rho^B - 2)$$

This can easily be positive while still satisfying (21) (and CSC). For example if $\rho^A = \frac{1}{3}$ and $\rho^B = 2$ both of satisfied. Therefore the socially optimal price level is strictly above cost in this case.

I believe there exists, but have not yet been able to construct, an example in which the socially optimal price level is strictly above the monopoly optimal price level. The reason is that this is not possible in the constant pass-through class and the most natural examples outside this class violate the second-order condition (20). So I here content myself with exhibiting a case in which, locally, a rise in the price level benefits consumer and therefore, because consumers are not directly harmed by an increase in cost, a rise in cost may benefit

consumers.

Suppose that demand on side B of the market is constant pass-through (see Weyl (2008b) for more details on these demand functions) with $\rho^B = 3$ (exploding at a price of 0) and demand on side A of the market is constant pass-through with $\rho^A = \frac{3}{10}$ (with no demand above price 0) for prices at or below -6 and demand jumps to 0 at price -6 . This is discontinuous, but still satisfies CSC as $\rho = 0$ if demand discontinuously drops to 0 (this can be seen as the limit of constant pass-through demands with pass-through approaching 0; a formal proof of this, as well as an example with no discontinuities, is available on request). Simple calculations show that with these demand functions and a cost of 1 the monopolist finds it optimal to charge $p^A = -6$ and $p^B = 21$. Note that at these prices consumers on side A earn no surplus and therefore its average surplus is 0. Consumer surplus on side A is market power (14, the same as $\bar{p} - c$) multiplied by the pass-through rate of 3. Substituting these into expression (18) yields a positive number. Thus social (and therefore consumer, as the price level is optimal for the monopolist and therefore moving it is neutral with respect to profits) welfare increases as the price level does at these prices. Furthermore because consumers are not directly affected by an increase in cost, this means an increase in cost may benefit them (through its tendency to increase the price level). Simple algebra, along these lines and available on request, also shows that social welfare increases with cost, so even taking into account the directly negative effects of an increase in cost, a rise in cost may be beneficial.

Note that all of these results also demonstrate the relevant possibilities under duopoly, because duopoly may mimic monopoly.

F Proof of Proposition 10

Consider the introduction of a differential amount of unbalanced competition on side A of the market; by the same reasoning as in the proof of Proposition 4 above if I can show that this

differential amount of competition can have a negative effect consumer welfare, this suffices to establish that there is a (non-differential) competition that is harmful to consumer welfare. The effect of this differential competition has sign, from equations (7) and (8)

$$-\rho^A D^{A'} B^B + \rho^A (1 - \rho^B) D^{A'} D^B + \rho^A \rho^B D^{A'} V^A$$

Note that this holds under monopoly for the effects of competition only; however clearly for the effects of a unilateral price control we can just divide by ρ^A , which does not change the sign, and under competition while the formula is not *necessary* (nor are those below), they always *may* occur if competition is balanced and therefore preserves $\mu^A = \mu^B$ governing price balance. Thus they are sufficient to show the possibility I want to demonstrate. Under monopoly and possibly under competition, this expression has the same sign as

$$\rho^A \bar{\rho}^A + \rho^A (1 - \rho^B) - \rho^A \rho^B \bar{\rho}^A$$

Suppose that demand on side A of the market is constant pass-through with $\rho^A = \frac{2}{3}$ and maximum price of 0 and that demand on side B is constant pass-through with $\rho^B = \frac{2}{3}$ and maximum price of 0 for prices below -1.8 and 0 above the price of -1.8 . If cost is -5 optimal prices are -2 to both sides. Average surplus on side B is .0903 and on side A is $\frac{2}{3}$. Plugging this into the above expression clearly yields a negative number, thus competition can harm consumer welfare under total cost-absorption. Because competition always harms firm profits, it can also harm social welfare. A case like this where A -side is cost amplifying is even easier to construct and I omit it here, but it is available on request.